CGC International Journal of Contemporary Technology and Research
ISSN: 2582-0486 (online) Vol.-2, Issue-2 DOI: 10.46860/cgcijctr.2020.06.26.126

Relevance of Matrices and Image Processing

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***Abstract*: An image is formed by the small bit of information namely pixel which is stored in the form of an array or a matrix. An image is converted into a digital form and some operations are carried out on this so that an improved image can be obtained and some specific information regarding the same can be recovered too; this procedure is known as image processing. Different processes in image processing involve different methods and operations applied. Matrix Theory has great importance in the operations applied in image processing. This manuscript is focused on the relationship of matrix theory and image processing in various applications of image processing. In order to understand the higher dimension matrices in image processing, some applications are considered to give a good insight.**

***Keywords:* Matrix, Image Processing, Compression, Watermarking, Document forgery**

I. Introduction

For today era of fast life, everyone wants the latest technologies to carry out their work in just few seconds or few minutes. This leads to the invention of new devices and advancement in technologies which can be used in our day to day life for convenience.

Image processing, being very effective and useful is not just limited to create an enhanced image, rather it has wide range of applications in various fields and technologies such as face detection, signature detection, fingerprint detection, license plate detection, medical diagnosis, robotics, remote sensing (aerial and satellite image interpretations), X-ray and cellular images, industry product sorting etc. [1]. Image processing is a technique which applies some operations on images to make that image better or to obtain some knowledge from that image. In Image processing, basically the following steps are involved as shown in figure 1[2].

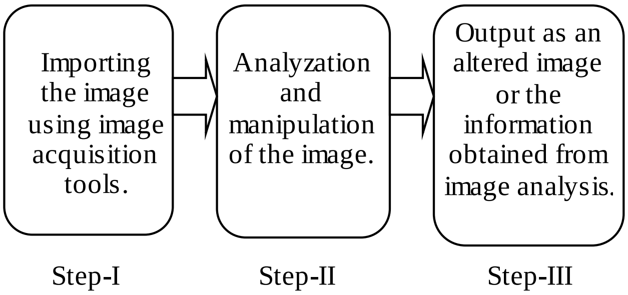


Fig. 1. Steps of Image processing

The operations involved in this technique includes some mathematical operations specially matrix operations. Firstly, all the images are considered as a 2D matrix with rows and column entries corresponding to the pixels of that image [3]. An image is not only can be expressed in matrix, but a matrix (especially high dimensional matrices) can also be represented by a corresponding image [4]. The intensity values of a matrix corresponding to an image usually ranges from 0-255 for 8-bit image which is called grey-scaled image. If the content of an image is stored in m x n x 3 matrix then it is called color image. Here 3 are used to indicate the corresponding values of Red (R), Green (G) and Blue (B). Each matrix specifies the amount of R, G and B that makes up the image [5]. In colored system, it is feasible to represent $256^3 = 2^{24} = 16777216$ distinct colors. Mainly two methods: Digital Image Processing and Analog Image Processing are used in Image Processing. For example: -The following image 2(a) of size 16x16 can be expressed in the matrix 2(b).

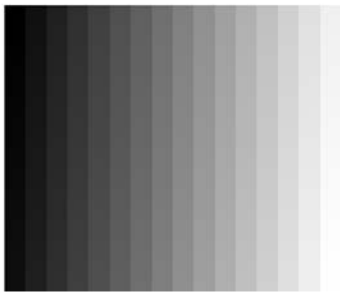


Fig. 2(a). Grey-scaled images

0	16	32	48	64	80	96	112	128	144	160	176	192	208	224	240
1	17	33	49	65	81	97	113	129	145	161	177	193	209	225	241
2	18	34	50	66	82	98	114	130	146	162	178	194	210	226	242
3	19	35	51	67	83	99	115	131	147	163	179	195	211	227	243
4	20	36	52	68	84	100	116	132	148	164	180	196	212	228	244
5	21	37	53	69	85	101	117	133	149	165	181	197	213	229	245
6	22	38	54	70	86	102	118	134	150	166	182	198	214	230	246
7	23	39	55	71	87	103	119	135	151	167	183	199	215	231	247
8	24	40	56	72	88	104	120	136	152	168	184	200	216	232	248
9	25	41	57	73	89	105	121	137	153	169	185	201	217	233	249
10	26	42	58	74	90	106	122	138	154	170	186	202	218	234	250
11	27	43	59	75	91	107	123	139	155	171	187	203	219	235	251
12	28	44	60	76	92	108	124	140	156	172	188	204	220	236	252
13	29	45	61	77	93	109	125	141	157	173	189	205	221	237	253
14	30	46	62	78	94	110	126	142	158	174	190	206	222	238	254
15	31	47	63	79	95	111	127	143	159	175	191	207	223	239	255

Fig. 2(b). Matrix corresponding to grey-scaled image in2(a)

From the above example, we can say that matrices form the basis of digital images. In order to solve the practical engineering problems reliably and efficiently, matrices have been proven to be a great mathematical tool. It has

ISSN: 2582-0486 (online) Vol.-2, Issue-2 DOI: 10.46860/cgcijctr.2020.06.26.126

major applications in wireless communications and signal processing [6]. Mostly, standard neural network is found to be effective and useful as the structure of their input is in the form of 1- dimensional vector. Whereas, in image processing, matrices are used as the input. These matrices are of 2- dimensions or higher dimensions. In case of input as higher dimensions, the data is converted into vectors before using in standard neural networks [7].

One of popular decomposition method which can be applied to all kinds of matrices is singular value decomposition (SVD) hence finds wide range of applications throughout the image processing techniques. SVD technique is widely used in hybridization of transform by researchers. But we cannot apply SVD alone on an image because of two important reasons. First reason is; it involves exhaustive computations involved when SVD is applied alone and second reason is; the problem of false positive in SVD. For these two reasons, it is preferred to implement SVD in hybridization with other transforms [8, 9]. The problem of detecting circular objects in cluttered image can be solved with modified Hough transform. This transform produces results in less computational time without requiring additional hardware [10].

II. APPLICATIONS

The very first application of digital image was witnessed when sent between London and New York through submarine cable. Later, development started in enhancing the visual quality of digital images by performing the various operations on matrices. Today, almost every area of technical field is impacted in one or other way by image processing. There are various applications of image processing but for understanding the role of matrices in image processing, few are considered below:

A. Image grey scrambling and pixel location Scrambling

For pixel location scrambling and image grey scrambling, a T-matrix known as Transformation matrix can be used [11]. For image grey Scrambling, let us consider the following notations:

P = A Grey Scaled Image

V = Number of Grey level in the image

W = Set of consecutive pixels with n distinct Grey Values.

i = Iteration number

Thus, Arnold Transform is as follows:

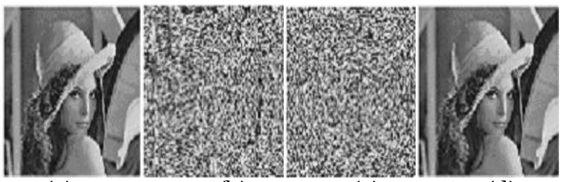
$$W_i = T^{i}_{n\times n}\, W_0 \,(\text{mod } V),\ i = 0, 1, 2, 3 \qquad \ldots.\ (1)$$

If the values of all pixels in P are selected by G, the image P will be scrambled to image P'. The image P' will be the scrambled image of image P if W selects the values of all the pixels in P.

Digital image encryption:

By taking the values as n = 3 and V = 256

With the use image grey scrambling, the result of an image encryption is shown in the following figure 3.



(a) (b) (c) (d)

Fig. 3. Image Encryption

Digital image encryption Result

a) Original image (256×256)

b) Encrypted image (i=2)

c) Encrypted image (i=100)

d) Recovered image (i=896)

Advantage of T-Matrix over Arnold Matrix: -

The image can be recovered when i will be equal to the period. The encryption of a digital image is much better with the use of T-Matrix than the Arnold Matrix because the period of Former is twice as long as the latter.

B. Image Compression

Matrices are used to represent the images in digital image processing. For example, the gray-scale image and its matrix representation are shown below in Figure 4 and in Figure 5. In a matrix, an element of a matrix is known as "pixel". Most digital images use integer numbers between 0 (to indicate black) and 255 (to indicate white) to represent the values of pixels, as a result, here are total 28 = 256 gray levels. In case of Color images, three matrices are used to represent the image. These matrices describe the amount of red (R), green (G) and blue (B) present in the image. Such a systematic representation is known as the RGB system [6].

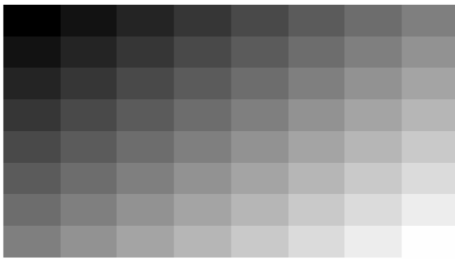


Fig.4. Gray-Scale Image

0	50	100	150	200	250	300	350
50	100	150	200	250	300	350	400
100	150	200	250	300	350	400	450
150	200	250	300	350	400	450	500
200	250	300	350	400	450	500	550
250	300	350	400	450	500	550	600
300	350	400	450	500	550	600	650
350	400	450	500	550	600	650	700

Fig.5- Matrix Representation of Gray-Scale Image

For solving, matrix element values are scaled in order to be compress into interval [0, 255] and a factor (255/maximum value) is multiplied by all matrix values. In the present example, this factor is (255/700) [4].

C. Matrices in Computer Graphics

In order to manipulate and construct a realistic animation of a polygonal figure some mathematical tools are being used in video gaming industry, which of them one component is matrix. The images are translated using linear transformations; these are the basis of the computer graphics where this process is done by using matrices. Square matrices are more reliable to represent linear transformation of objects. Three Dimensional images are projected to two dimensional planes using these matrices. In Graphics, the first step is to represent the digital image in a matrix whose rows and columns are rows and columns of pixels and the numerical entries are the color values of the pixels. In video game Graphics, matrices are used to manipulate a point is ordinary mathematical approach [12].

D. Digital image processing and operations with matrices

The image Transition Effect which is used in slide shows and PPT's can be created by using the operations like sum of matrices and multiplication of matrix by a scalar. More accurately, suppose two same sized grayscale images, denoted by the matrices X and Y. For each real number scalar k in the interval [0, 1], let us define the matrix $S(t) = (1 - t) X + t Y$. Note that $S(0) = X$, $S(1) = Y$ and the entries of the matrix S(t) for each $0<t<1$, are between the entries of the matrices X and Y. Thus, for $0 < t < 1$, the matrix S(t) varies from X to Y. For color images, the above transformation must be applied to the matrices R, G and B that make each image.

E. Images corresponding to matrices

By presenting 0 value as black and maximum positive value as white, in the matrices having zero or positive entries, all positive matrix values can be presented as grayscale values. By giving negative values, the nuances from black to yellow and positive values from black to white, he matrices with negative and positive entries can be presented. For example, the image of the matrix in the figure 6 can be presented as shown in figure 7.

$$\begin{bmatrix} -1 & 1 & 0 & -1 & 1 & 0 & -1 & 1 \\ 1 & 0 & -1 & 1 & 0 & -1 & 1 & 0 \\ 0 & -1 & 1 & 0 & -1 & 1 & 0 & -1 \\ -1 & 1 & 0 & -1 & 1 & 0 & -1 & 1 \\ 1 & 0 & -1 & 1 & 0 & -1 & 1 & 0 \\ 0 & -1 & 1 & 0 & -1 & 1 & 0 & -1 \\ -1 & 1 & 0 & -1 & 1 & 0 & -1 & 1 \\ 1 & 0 & -1 & 1 & 0 & -1 & 1 & 0 \end{bmatrix}$$

Fig.6. Matrix with zero and negative entries

Similarly, we can present pure imaginary matrices by presenting positive imaginary entries from black to red and Negative imaginary entries from black to green [4]. For example, the matrix in figure 8 can be presented by the image shown in figure 9.

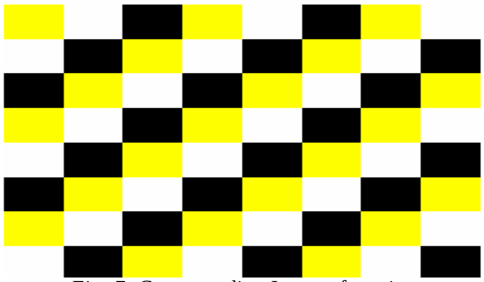


Fig. 7. Corresponding Image of matrix

$$\begin{bmatrix} +4i & +4i & +4i & +4i & +4i & +4i & +4i & +4i & +4i \\ +3i & +3i & +3i & +3i & +3i & +3i & +3i & +3i & +3i \\ +2i & +2i & +2i & +2i & +2i & +2i & +2i & +2i & +2i \\ +1i & +1i & +1i & +1i & +1i & +1i & +1i & +1i & +1i \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1i & -1i & -1i & -1i & -1i & -1i & -1i & -1i & -1i \\ -2i & -2i & -2i & -2i & -2i & -2i & -2i & -2i & -2i \\ -3i & -3i & -3i & -3i & -3i & -3i & -3i & -3i & -3i \\ -4i & -4i & -4i & -4i & -4i & -4i & -4i & -4i & -4i \end{bmatrix}$$

Fig. 8- matrix with all complex entries

Fig. 9. Image of corresponding matrix

F. Toeplitz Matrix in Watermarking

The efficient and well – liked method for dispiriting the delivery of copyrighted digital image information and illegal copying is digital watermarking, which is a good solution for authentication of images. To represent watermarking pattern a square matrix known as Diagonal constant matrix or Toeplitz matrix is defined in which from left to right each descending diagonal is constant [13].The matrix has been named after Otto Toeplitz.

Consider a vector with 9 elements as

$$M = \{p, q, r, s, t, u, v, w, x\}.$$

Corresponding to this vector a 5x5 ordered Toeplitz matrix can be constructed, first five elements of the vector are the elements of first row of the matrix and in the first column of the matrix except the first element all the entries are last four elements of the vector. The other entries of the matrix can be found as given in equation (1).The matrix is represented in figure 10.

ISSN: 2582-0486 (online) Vol.-2, Issue-2 DOI: 10.46860/cgcijctr.2020.06.26.126

$$T = \begin{bmatrix} p & q & r & s & t \\ u & p & q & r & s \\ v & u & p & q & r \\ w & v & u & p & q \\ x & w & v & u & p \end{bmatrix}$$

$$A\ (k, j) = A\ (k-1, j-1) \qquad \text{…. (2)}$$

As Toeplitz matrix is a square matrix, thus the images of equal rows and columns are considered. For example, In Figure 14, the size of original image has been taken as 512 x512. Then from original image, a watermarked signal as a Toeplitz matrix with the size 256x256 has constructed and embedded within itself. Even after embedding watermark high image quality is preserved and also there is no visual difference. In the following figure, image 10(a) is the original image, image 10(b) is the watermarked image and image 10(c) is the difference image [12].

10(a) 10(b) 10(c)

Fig. 10. Image Watermarking

G. Detection of Image and Document forgery

For the purpose to check the authenticity of the image, a method based on Energy Deviation Measure can be used. This method has great advantage of being performed with fewer dimensions and not relying on quality factor of Image Compression [14].

Due to Image forgery has become common, it is necessary to check authenticity of the image using some forensics techniques and tools. A blind JPEG Classifier using noise and edge characteristics of CCC (Color Choice Characteristics) can also be implemented for the detection of forged images. In spite of having the advantage of being performed with lesser dimensions and high accuracy of the classifier, it has the limitation of giving slightly minimum accuracy for Spliced images [15].

The detection of Printed Document forgery can be done by a computational approach which uses the features of extraction methodology like ORB (ORIENTED FAST ROTATED BRIEF) and SURF (SPEEDED UP ROBUST FEATURES). Using this the resource printer of the printed document can be detected [16].

H. Detection of Cracks

With the use of Image Processing in various techniques the crack and its depth can be identified automatically. Automatic crack detection is more reliable upon the manual detection procedures. The reason of more reliability of automatic detection by image processing is the accurate results it provides. Crack Detection using image processing follows the steps shown in the figure 11:

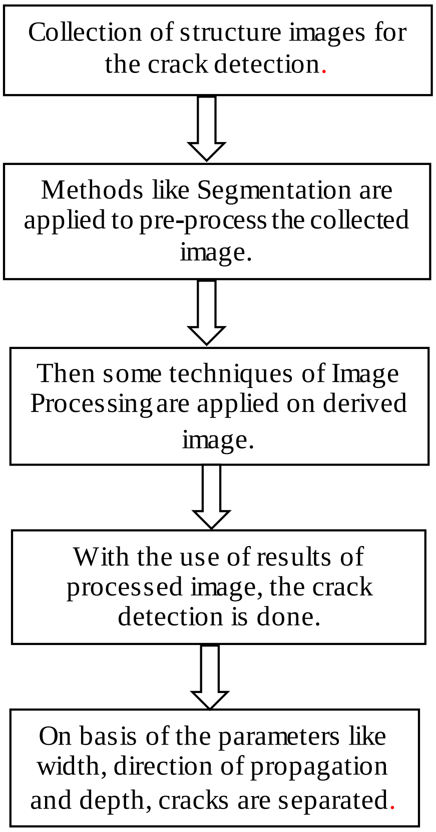


Fig. 11. Steps of crack detection

Based on the types of images such as Infrared image, camera image, Ultrasonic image, Laser Image etc. the detected cracks are separated [17].

I. Face Recognition

Programmed design for face recognition involves many functions, including matrix multiplication, matrix transposition, mean calculation, covariance matrix, eigen value determination, feature vectors, projection and recognition. Face Recognition has become very important identification method. This is used in our daily life as in mobiles, biometrics, e-passports etc. Image processing on the basis of different algorithms identifies the human faces. It can not only detect a single face but it can also detect the multiple faces at the same time. With the use of the inbuilt algorithms in open CV like PCA, LDA, LBPH, a robot identify the faces on real time basis [18]. Face recognition outcome depend on features that are obtained to characterize the face pattern and classification methods used to differentiate among faces whereas face localization and normalization are the basis for obtaining effective features. The basic process of face recognition is shown in figure 12.

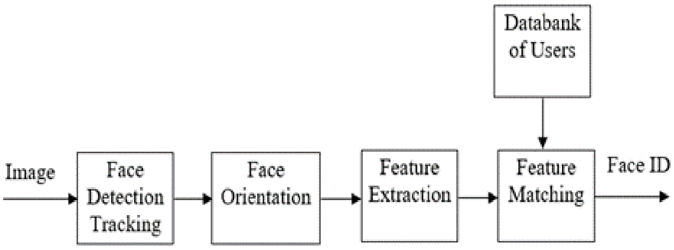


Fig. 12. Flow diagram of face recognition

III. Conclusion

The images in digital image processing are enhanced by operating on corresponding matrix of the image. In this paper we have reviewed the considerable role of matrices in processing and representing digital images. This paper also reviewed the utility of matrices in many areas like data compression, image compression, computer graphics, digital image processing, crack detection, embedding watermark etc. It is also concluded that the two main claim areas in which researchers can emphasis are enhancement of visual information for human interpretation and processing of image information for storage, communication and representation for self-directed machine perception. The future of image processing includes evolution in various digital image processing applications.

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